

Non-minimally Coupled Quintom Model Inspired by String Theory

J. Sadeghi ^{a*} M. R. Setare ^{b†}, A. Banijamali ^{a‡} and F. Milani ^a

^a *Sciences Faculty, Department of Physics, Mazandaran University,*

P.O.Box 47415-416, Babolsar, Iran

^b *Department of Science, Payame Noor University, Bijar, Iran*

April 3, 2008

Abstract

In this paper we consider a quintom model of dark energy with a single scalar field T given by a Lagrangian which inspired by tachyonic Lagrangian in string theory. We consider non-minimal coupling of tachyon field to the scalar curvature, then we obtain the equation of state (EoS), and the condition required for the model parameters when ω crosses over -1 .

Keywords: Quintom model; Tachyon; Non-minimal coupling.

*Email: pouriya@ipm.ir

†Email: rezakord@ipm.ir

‡Email: abanijamali@umz.ac.ir

1 Introduction

Many cosmological observations, such as SNe Ia [1], WMAP [2], SDSS [3], Chandra X-ray observatory [4] etc., discover that our universe is undergoing an accelerated expansion. They also suggest that our universe is spatially flat, and consists of about 70 % dark energy (DE) with negative pressure, 30 % dust matter (cold dark matter plus baryons), and negligible radiation. Dark energy has been one of the most active fields in modern cosmology [5].

In modern cosmology of dark energy, the equation of state parameter (EoS) $\omega = \frac{p}{\rho}$ plays an important role, where p and ρ are its pressure and energy density, respectively. To accelerate the expansion, the EoS of dark energy must satisfy $\omega < -\frac{1}{3}$. The simplest candidate of the dark energy is a tiny positive time-independent cosmological constant Λ , for which $\omega = -1$. However, it is difficult to understand why the cosmological constant is about 120 orders of magnitude smaller than its natural expectation (the Planck energy density). This is the so-called cosmological constant problem. Another puzzle of the dark energy is the cosmological coincidence problem: why are we living in an epoch in which the dark energy density and the dust matter energy are comparable?. As a possible solution to these problems various dynamical models of dark energy have been proposed, such as quintessence [6, 7]. The analysis of the properties of dark energy from recent observations mildly favor models with ω crossing -1 in the near past. So far, a large class of scalar-field dark energy models have been studied, including tachyon [8], ghost condensate [9, 10] and quintom [11, 12, 13, 14], and so forth. In addition, other proposals on dark energy include interacting dark energy models [15], braneworld models [16], and holographic dark energy models [17], etc. The Ref.[11] is the first paper showing explicitly the difficulty of realizing ω crossing over -1 in the quintessence and phantom like models. Because it has been proved [11, 18, 19] that the dark energy perturbation would be divergent as the equation of state ω approaches to -1. The quintom scenario of dark energy is designed to understand the nature of dark energy with ω across -1. The quintom models of dark energy differ from the quintessence, phantom and k-essence and so on in the determination of the cosmological evolution and the fate of the universe.

To realize a viable quintom scenario of dark energy it needs to introduce extra degree of freedom to the conventional theory with a single fluid or a single scalar field. The first model of quintom scenario of dark energy is given by Ref.[11] with two scalar fields. This model has been studied in detail later on [12, 13, 14] (to see the bouncing solution in the universe dominated by quintom matter refer to [20]). Recently there has been an upsurge in activity for constructing such model in string theory [21]. In the context of string theory, the tachyon field in the world volume theory of the open string stretched between a D-brane and an anti-D-brane or a non-BPS D-brane plays the role of scalar field in the quintom model [22]. The effective action used in the study of tachyon cosmology consists of the standard Einstein-Hilbert action and an effective action for the tachyon field on unstable D-brane or D-brane anti D-brane system. What distinguishes the tachyon action from the standard Klein- Gordon form for scalar field is that the tachyon action is non-standard and is of the " Dirac-Born-Infeld " form [23]. The tachyon potential is derived from string theory itself and has to satisfy some definite properties to describe tachyon condensation and other

requirements in string theory.

In this paper, we consider an action for tachyon non-minimally coupled to gravity [24] inspired by the string theory. An outline of this paper is as follows. In section 2 we introduce action for tachyon non-minimally coupled to gravity with an extra term. By performing a conformal transformation we obtain the new action. In order to discuss the equation of state we derive the corresponding energy density and pressure for this model. By solving this equation we obtain the conditions required for the ω across -1. Section 3 is devoted to discussion of our results.

2 Non-minimally coupled tachyon gravity with extra term

We consider the action Ref.[25] for tachyon non-minimally coupled to gravity, in this action we add an extra term $T\Box T$ to the usual terms in the square root. In that case the following action is the same as Ref.[26] just different to the $Rf(T)$,

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} Rf(T) - AV(T) \sqrt{1 - \alpha' g^{\mu\nu} \partial_\mu T \partial_\nu T + \beta' T \Box T} \right], \quad (1)$$

where $f(T)$ is a function of the tachyon T and corresponds to the non-minimal coupling factor. Here $V(T)$ is the tachyon potential which is bounded and reaching its minimum asymptotically. $M_P = \frac{1}{\sqrt{8\pi G}}$ is reduced Planck mass.

Action (1) generalizes the usual "Born- Infeld- type" action for the effective description of tachyon dynamics which can be obtained by the stringy computations for a non- BPS D3- brane in type II theory. The extra term in action (1) has a significant cosmological consequence, so we cannot exclude its existence in an action such as the "Born- Infeld- type" action.

The model with operator $T\Box T$ for realizing of ω crossing -1 has been proposed in [12]. They considered a dimension-6 operator as $(\Box T)^2$. However in the present paper, the operator $T\Box T$ appears at the same order as the operator $\partial_\mu T \partial^\mu T$ does in the "Born- Infeld- type" action and also we take into account scalar curvature non-minimally coupled to the tachyon field.

The action (1) can be brought to the simpler form to derive the equation of motion, energy density and pressure, by performing a conformal transformation as follows:

$$g_{\mu\nu} \longrightarrow f(T)g_{\mu\nu}. \quad (2)$$

The above conformal transformation yields to the following action:

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} \left(R - \frac{3}{2} \frac{f'^2}{f^2} \partial_\mu T \partial^\mu T \right) - A\tilde{V}(T) \sqrt{1 - (\alpha' f(T) - 2\beta' f'(T)T) \partial_\mu T \partial^\mu T + \beta' f(T) T \Box T} \right] \quad (3)$$

where $\tilde{V}(T) = \frac{V(T)}{f^2}$ is now the effective potential of the tachyon.

For a flat Friedman- Robertson- Walker (FRW) universe and a homogenous scalar field T , the equation of motion can be solved equivalently by the following two equations,

$$\ddot{\psi} + 3H\dot{\psi} = \left(\frac{2\beta'f'T - \alpha'f}{fT}\right)\dot{\psi}\dot{T} - \frac{A^2\beta'f\tilde{V}T}{2\psi}\tilde{V}' - \frac{3M_P^2}{2}\left(\frac{ff'f'' - f'^3}{f^3}\right)\dot{T}^2 - \left[\frac{(1-\beta')(\alpha' - 2\beta')}{\beta'}\frac{f'}{f} - \frac{\alpha'}{T}\right]\frac{\psi\dot{T}^2}{T}, \quad (4)$$

$$\ddot{T} + 3H\dot{T} = \frac{2\left[\left(\frac{ff''+\beta'f'}{f^2}\right)T\dot{T}^2 - 2(\alpha' - 2\beta'\frac{f'}{f}T)H\dot{T}\right]}{1 + \frac{2\alpha'}{\beta'} - 3\frac{f'}{f}T - \frac{3M_P^2}{2}\left(\frac{f'}{f}\right)^2\frac{T}{\psi}}, \quad (5)$$

where

$$\psi = \frac{\partial\mathcal{L}}{\partial\Box T} = -\frac{A\beta'\tilde{V}fT}{2h}$$

$$h = \sqrt{1 - (\alpha'f - 2\beta'f'T)\partial_\mu T\partial^\mu T + \beta'fT\Box T}$$

and

$$\tilde{V}' = \frac{d\tilde{V}}{dT}.$$

$H = \frac{\dot{a}}{a}$ is the Hubble parameter.

The energy momentum tensor $T^{\mu\nu}$ is given by the standard definition:

$$\delta_{g_{\mu\nu}}S = - \int d^4x \frac{\sqrt{-g}}{2} T^{\mu\nu} \delta g_{\mu\nu}.$$

So the energy density, pressure and Friedman equation are found to be

$$\rho = A\tilde{V}h + \frac{d}{a^3dt}(a^3\psi\dot{T}) + (\alpha'f - 2\beta'f'T)\frac{A\tilde{V}}{h}\dot{T}^2 - 2\dot{\psi}\dot{T} + \frac{3M_P^2}{4}\left(\frac{f'^2}{f}\right)\dot{T}^2, \quad (6)$$

$$p = -A\tilde{V}h - \frac{d}{a^3dt}(a^3\psi\dot{T}) + \frac{3M_P^2}{4}\left(\frac{f'^2}{f}\right)\dot{T}^2, \quad (7)$$

$$H^2 = \frac{A}{3M_P^2}\tilde{V}h + \frac{d}{3M_P^2a^3dt}(a^3\psi\dot{T}) + \frac{(\alpha'f - 2\beta'f'T)}{3M_P^2}\frac{A\tilde{V}\dot{T}^2}{h} - \frac{2}{3M_P^2}\dot{\psi}\dot{T} + \frac{1}{4}\left(\frac{f'^2}{f}\right)\dot{T}^2. \quad (8)$$

We now study the cosmological evolution of equation of state for the present model. The equation of state is $p = \omega\rho$. To explore the possibility of the ω across -1, we have to check

$\frac{d}{dt}(\rho + p) \neq 0$ when $\omega \longrightarrow -1$.

From equations (6) and (7) one can obtain the following expressions,

$$\rho + p = \frac{3M_P^2}{2} \left(\frac{f'^2}{f} \right) \dot{T}^2 + (\alpha' f - 2\beta' f' T) \frac{\tilde{V}}{h} \dot{T}^2 + A\beta' \frac{d}{dt} \left(\frac{f\tilde{V}T}{h} \right) \dot{T}, \quad (9)$$

and

$$\begin{aligned} \frac{d}{dt}(\rho + p) &= \left[3M_P^2 \left(\frac{f'^2}{f} \right) \dot{T} + A\beta' \frac{d}{dt} \left(\frac{f\tilde{V}T}{h} \right) \right] \ddot{T} + \left[(\alpha' \dot{f} - 2\beta' f') \frac{\tilde{V}}{h} + 3M_P^2 \frac{f'}{f} \frac{d}{dt} \left(\frac{f'}{f} \right) \right] \dot{T}^2 \\ &- 2\beta' f' \frac{\tilde{V}}{h} \dot{T}^3 + (\alpha' f - 2\beta' f' T) \frac{d}{dt} \left(\frac{\tilde{V}\dot{T}^2}{h} \right) + A\beta' \frac{d^2}{dt^2} \left(\frac{f\tilde{V}T}{h} \right) \dot{T}. \end{aligned} \quad (10)$$

From equation (9) one finds $\dot{T} = 0$, or $\left[\frac{3M_P^2}{2} \left(\frac{f'^2}{f} \right) + (\alpha' f - 2\beta' f' T) \frac{\tilde{V}}{h} \right] \dot{T} = -A\beta' \frac{d}{dt} \left(\frac{f\tilde{V}T}{h} \right)$ when $\omega \longrightarrow -1$.

If $\dot{T} = 0$ then equation (10) gives,

$$\frac{d}{dt}(\rho + p) = A\beta' \frac{d}{dt} \left(\frac{f\tilde{V}T}{h} \right) \ddot{T} = -\frac{\tilde{V}f^2}{2h^3} \beta'^2 T^2 \ddot{T} \frac{d}{dt} \square T. \quad (11)$$

So we need $T \neq 0$, $\ddot{T} \neq 0$ and $\frac{d}{dt} \square T \neq 0$, for having ω across -1. With these crossing over condition and asymptotically behavior of $V(T)$ one concludes that crossing over -1 must happen before reaching the potential minimum asymptotically. This implies that when crossing over -1 occur the field T must continue to run away as it should be since we have $\ddot{T} \neq 0$. One can obtain the same result with the second condition and also note that in the second case \dot{T} is non-zero.

Now we consider two specific examples to see how EoS evolves in our model. In numerical calculations we have used the expansion of $V(T) = e^{-\lambda T^2}$ (motivated from boundary string field theory [26]) in Figure 1 and $V(T) = \frac{V_0}{e^{\lambda T} + e^{-\lambda T}}$ in Figure 2, also we take $f(T) = 1 + \sum_{i=1} c_i T^{2i}$.

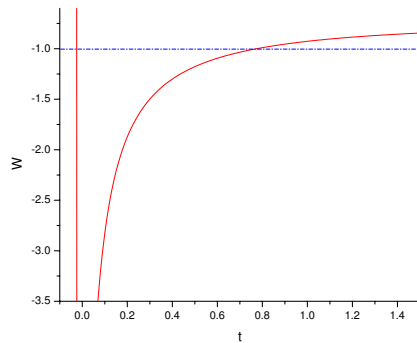


Figure 1: The plot of EoS for the potential $V(T) = e^{-\lambda T^2}$, $\alpha = -2$ and $\beta = 2.2$. Initial values are $\phi = 1$ and $\dot{\phi} = 3$.

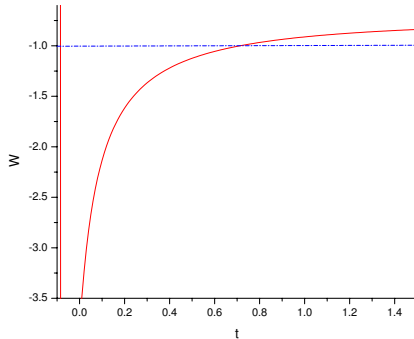


Figure 2: The plot of EoS for the potential $V(T) = \frac{V_0}{e^{\lambda T} + e^{-\lambda T}}$, $\alpha = -2$, $\beta = 2.2$, $\lambda = 2$ and $V_0 = 5$. Initial values are $\phi = 1$ and $\dot{\phi} = -3$.

3 Conclusion

In order to solve cosmological problems and because the lack of our knowledge, for instance to determine what could be the best candidate for DE to explain the accelerated expansion of universe, the cosmologists try to approach to best results as precise as they can by considering all the possibilities they have. Within the different candidates to play the role of the dark energy, the quintom model, has emerged as a possible model with EoS across -1 . In this paper we have introduced a string inspired quintom model non-minimally coupled to gravity with an extra term in the usual effective action of tachyon dynamics. This modification has been done due to crossing over -1 of EoS. We showed that crossing over -1 occur in this model, before reaching the tachyon potential asymptotically to its minimum.

References

- [1] A. G. Riess et al. [Supernova Search Team Collaboration], *Astrophys. J.* **607**, 665 (2004) [astro-ph/0402512]; R. A. Knop et al., [Supernova Cosmology Project Collaboration], *Astrophys. J.* **598**, 102 (2003) [astro-ph/0309368]; A. G. Riess et al. [Supernova Search Team Collaboration], *Astron. J.* **116**, 1009 (1998) [astro-ph/9805201]; S. Perlmutter et al. [Supernova Cosmology Project Collaboration], *Astrophys. J.* **517**, 565 (1999) [astro-ph/9812133].
- [2] C. L. Bennett et al., *Astrophys. J. Suppl.* **148**, 1 (2003) [astro-ph/0302207]; D. N. Spergel et al., *Astrophys. J. Suppl.* **148**, 175 (2003) [astro-ph/0302209].
- [3] M. Tegmark et al. [SDSS Collaboration], *Phys. Rev. D* **69**, 103501 (2004) [astro-ph/0310723]; M. Tegmark et al. [SDSS Collaboration], *Astrophys. J.* **606**, 702 (2004) [astro-ph/0310725]; U. Seljak et al., *Phys. Rev. D* **71**, 103515 (2005) [astro-ph/0305162].

- (2005) [astro-ph/0407372]; J. K. Adelman-McCarthy et al. [SDSS Collaboration], astro-ph/0507711; K. Abazajian et al. [SDSS Collaboration], astro-ph/0410239; astro-ph/0403325; astro-ph/0305492;
- [4] S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian and L. van Speybroeck, Mon. Not. Roy. Astron. Soc. **353**, 457 (2004) [astro-ph/0405340].
- [5] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003)[astro-ph/0207347]; T. Padmanabhan, Phys. Rept. **380**, 235 (2003) [hep-th/0212290]; S. M. Carroll, astro-ph/0310342; R. Bean, S. Carroll and M. Trodden, astro-ph/0510059; V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D **9**, 373 (2000) [astro-ph/9904398]; S. M. Carroll, Living Rev. Rel. **4**, 1 (2001) [astro-ph/0004075]; T. Padmanabhan, Curr. Sci. **88**, 1057 (2005) [astro-ph/0411044]; S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989); S. Nobbenhuis, gr-qc/0411093.
- [6] R. R. Caldwell, R. Dave and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998) [astro-ph/9708069]; C. Wetterich, Nucl. Phys. B **302**, 668 (1988).
- [7] P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D **59**, 123504 (1999) [astro-ph/9812313].
- [8] A. Sen, JHEP **0207**, 065 (2002) [hep-th/0203265]; T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002) [hep-th/0204150].
- [9] N. Arkani-Hamed, H. C. Cheng, M. A. Luty and S. Mukohyama, JHEP **0405**, 074 (2004) [hep-th/0312099].
- [10] F. Piazza and S. Tsujikawa, JCAP **0407**, 004 (2004) [hep-th/0405054].
- [11] B. Feng, X. Wang and X. Zhang, Phys. Lett. B **607**, 35 (2005).
- [12] M. Li, B. Feng, and X. Zhang, JCAP **0512**, 002 (2005); X.-F. Zhang , and T.-T. Qiu, Phys. Lett. B **642**, 187 (2006).
- [13] P. S. Apostolopoulos, and N. Tetradis, Phys. Rev. D **74**, 064021 (2006); H.-S. Zhang, and Z.-H. Zhu, Phys. Rev. D **75**, 023510 (2007).
- [14] B. Feng, M. Li, Y. Piao, and X. Zhang, Phys. Lett. B **634**, 101 (2006); X.-F. Zhang, H. Li, Y.-S. Piao, and X. M. Zhang, Mod. Phys. Lett. A **21**, 231 (2006); Z .K. Guo, Y. S. Piao, X. M Zhang, Y. Z. Zhang, Phys. Lett. B **608**, 177, (2005); Y. F Cai, H. Li, Y. S. Piao, X. M. Zhang, Phys. Lett. B **646**, 141, (2007); Z.-K. Guo, Y. -S. Piao, X. Zhang, Y. -Z. Zhang, Phys. Rev. D **74**, 127304, (2006).
- [15] L. Amendola, Phys. Rev. D **62**, 043511 (2000) [astro-ph/9908023]; M. Szydlowski, A. Kurek, and A. Krawiec Phys. Lett. B **642**, 171, (2006) [astro-ph/0604327]; M. R. Setare, Phys. Lett. B **642**, 1, (2006); M. R. Setare, Eur. Phys. J. **C50**, 991, (2007).

- [16] C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D **65**, 044023 (2002) [astro-ph/0105068];
M. R. Setare, Phys. Lett. **B642**, 421, (2006).
- [17] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Rev. Lett. **82**, 4971 (1999) [hep-th/9803132];
P. Horava and D. Minic, Phys. Rev. Lett. **85**, 1610 (2000) [hep-th/0001145];
E. Elizalde, S. Nojiri, S. D. Odintsov and P. Wang, Phys. Rev. D **71**, 103504 (2005) [hep-th/0502082];
M. R. Setare, Phys. Lett. B **644**, 99, (2007);
M. R. Setare, 01, 023, JCAP (2007);
M. R. Setare, J. Zhang, and X. Zhang, JCAP **0703**, 007, (2007);
M. R. Setare, Phys. Lett. **B648**, 329, (2007).
- [18] G.-B. Zhao, J.-Q. Xia, M. Li, B. Feng, and X. Zhang, Phys. Rev. D **72**, 123515 (2005)
- [19] R. R. Caldwell, M. Doran, Phys. Rev. D **72**, 043527 (2005); A. Vikman, Phys. Rev. D **71**, 023515 (2005); W. Hu, Phys. Rev. D **71**, 047301 (2005).
- [20] Y. -F. Cai, T. Qiu , Y.-S. Piao, M. Li, X. Zhang JHEP, 0710, 071, (2007).
- [21] F. Quevedo, Class. Quant. Grav. **19**, 5721 (2002), [hep-th/0210292].
- [22] S. Alexander, Phys. Rev. D **65**, 023507 (2002) [hep-th/0105032]; A. Mazumdar, S. Panda and A. Perez-Lorenzana, Nucl. Phys. B **614**, 101 (2001), [hep-ph/0107058]; G. Gibbons, Phys. Lett. B **537**, 1 (2002), [hep-th/0204008].
- [23] A. Sen, JHEP **9910**, 008 (1999), [hep-th/9909062]; E. Bergshoeff, M. de Roo, T. de Wit, E. Eyras and S. Panda, JHEP **0005**, 009 (2000) [hep-th/0003221]; J. Kluson, Phys. Rev. D **62**, 126003 (2000) [hep-th/0004106].
- [24] Y.-S. Piao, Q.-G. Huang, X. Zhang and Y.-Z. Zhang, Phys. Lett. B **570**, 1 (2003) [hep-th/0212219].
- [25] P. Chingangbam, S. Panda and A. Deshamukhya hep-th/0411210.
- [26] Y.-F Cai, M. LI, J.-X Lu, Y.-S Piao, T. Qiu and X. Zhang hep-th/0701016.